

Irreversible processes in a Universe modelled as a mixture of a Chaplygin gas and radiation

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Abstract

The evolution of a Universe modelled as a mixture of a Chaplygin gas and radiation is determined by taking into account irreversible processes. This mixture could interpolate periods of a radiation dominated, a matter dominated and a cosmological constant dominated Universe. The results of a Universe modelled by this mixture are compared with the results of a mixture whose constituents are radiation and quintessence. Among other results it is shown that: (a) for both models there exists a period of a past deceleration with a present acceleration; (b) the slope of the acceleration of the Universe modelled as a mixture of a Chaplygin gas with radiation is more pronounced than that modelled as a mixture of quintessence and radiation; (c) the energy density of the Chaplygin gas tends to a constant value at earlier times than the energy density of quintessence does; (d) the energy density of radiation for both mixtures coincide and decay more rapidly than the energy densities of the Chaplygin gas and of quintessence.

Key words: Chaplygin gas, accelerated Universe, quintessence

Recent measurements of the anisotropy of the cosmic microwave background and of the type Ia supernova SN 1997ff redshift indicate that the Universe is flat with a present positive acceleration and a past decelerating period [1, 2, 3]. Moreover, it has been recognised that a significant part of the energy density of the Universe is not due to matter or radiation but to an extraordinary non-baryonic matter and energy, which has a negative pressure and is the responsible for the present acceleration of the Universe. Two candidates for this energy, also called dark energy, are found in the literature. One of them is the quintessence with an equation of state $p_X = w_X \rho_X$ and with the condition $w_X < -1/3$ (see, for example [4, 5, 6]), while the other refers to a Chaplygin gas with an exotic

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equation of state $p_c = -A/\rho_c$ subjected to the condition $A > 0$ (see, for example [7, 8, 9, 10]). For some properties of the Chaplygin gas one is referred to the book by Jackiw [11].

The irreversible processes in a homogeneous and isotropic Universe are normally described by the so-called viscous cosmological models which are based on thermodynamic theories. There exist two kinds of thermodynamic theories: one within the framework of Eckart's (or first order) thermodynamic theory (see, for example [12, 13, 14, 15, 16]) where the non-equilibrium pressure is taken as a constitutive quantity, while the other considers an evolution equation for the non-equilibrium pressure within the framework of the extended (or causal or second order) thermodynamic theory (see, for example [15, 16, 17, 18, 19, 20, 21]).

Recently [16] the evolution of a Universe modelled as a mixture of a matter field with quintessence was analysed by taking into account the irreversible transfer of energy densities of the matter and gravitational fields. Among other results, it was shown that: (a) there exists a period of past decelerating followed by a present acceleration of the Universe due to the quintessence; (b) the energy density of quintessence decays more slowly than the energy density of the matter field does.

The objective of this work is to analyse a Universe modelled as a mixture of a Chaplygin gas and radiation by taking into account irreversible processes within the framework of extended thermodynamics. This mixture is more suitable than the one with radiation and quintessence as constituents, since - according to the work [7] - the Chaplygin gas can interpolate a matter dominated Universe (dust or pressure-less fluid) and a cosmological constant dominated Universe. Hence, the mixture we are interested in could interpolate a period of a radiation dominated, a matter dominated and a cosmological constant dominated Universe. This interpolation between the three periods is not possible for a Universe modelled by a mixture of radiation and quintessence. The only possibility is to adjust in each period the barotropic equation of state of the matter field.

In this work we compare the results of a Universe modelled by a mixture of a Chaplygin gas and radiation with the results of a mixture whose constituents are radiation and quintessence. Among other results it is shown that for both models there exists a period of a past deceleration with a present acceleration while the slope of the acceleration of the Universe modelled as a mixture of a Chaplygin gas with radiation is more pronounced than that modelled as a mixture of quintessence and radiation. Moreover, the energy density of the Chaplygin gas tends to a constant value at earlier times than the energy density of quintessence does and the energy density of radiation for both mixtures coincide and decay more rapidly than the energy densities of the Chaplygin gas and of quintessence.

We consider the Robertson-Walker metric and model a spatially flat,

homogeneous and isotropic Universe as a mixture of a radiation field and an exotic fluid characterised by the so-called Chaplygin gas. The pressure of the radiation field p_r and the pressure of the Chaplygin gas p_c are related, respectively, to their energy densities ρ_r and ρ_c by

$$p_r = \frac{1}{3}\rho_r, \quad p_c = -\frac{A}{\rho_c}, \quad \text{with} \quad A = \text{constant} > 0. \quad (1)$$

For this kind of Universe, the energy-momentum tensor $T^{\mu\nu}$ of the sources that appear in Einstein's field equations is given by

$$T^{\mu\nu} = (\rho_r + \rho_c + p_r + p_c + \varpi)U^\mu U^\nu - (p_r + p_c + \varpi)g^{\mu\nu}, \quad (2)$$

where U^μ (such that $U^\mu U_\mu = 1$) is the four-velocity and ϖ denotes the non-equilibrium pressure which is responsible for the dissipative effects during the evolution of the Universe.

The balance equation for the energy density of the mixture follows from the conservation law of the energy-momentum tensor $T^{\mu\nu}_{;\nu} = 0$ which in a comoving frame reads

$$\dot{\rho}_r + \dot{\rho}_c + 3H(\rho_r + \rho_c + p_r + p_c + \varpi) = 0. \quad (3)$$

Above, the over-dot refers to a differentiation with respect to time and $H = \dot{a}/a$ is the Hubble parameter with $a(t)$ denoting the cosmic scale factor.

The Friedmann equation connects the evolution of the cosmic scale factor with the energy densities of the radiation field and of the Chaplygin gas, i.e.,

$$H^2 = \frac{8\pi G}{3}(\rho_r + \rho_c), \quad (4)$$

where G is the gravitational constant.

We assume that the Chaplygin gas interacts only with itself and is minimally coupled to the gravitational field. In this case, the balance equation for the energy density of the Chaplygin gas decouples from the energy density of the mixture (3) and can be written as

$$\dot{\rho}_c + 3H(\rho_c + p_c) = 0. \quad (5)$$

From equations (3) and (5) we get that the balance equation for the energy density of the radiation field reduces to

$$\dot{\rho}_r + 3H(\rho_r + p_r) = -3H\varpi. \quad (6)$$

In order to interpret the above equation we follow the work [16] where the component of the energy-momentum pseudo-tensor of the gravitational

field T_G^{00} in a flat Robertson-Walker metric was identified with the energy density ρ_G of the gravitational field, i.e.,

$$T_G^{00} \equiv \rho_G = -\frac{3}{8\pi G} \left(\frac{\dot{a}}{a} \right)^2 = -(\rho_r + \rho_c). \quad (7)$$

The last equality on the right-hand side of (7) follows from the Friedmann equation (4). Hence, the differentiation of (7) thanks to (6) leads to:

$$\dot{\rho}_G + 3H(\rho_G - p_r - p_c) = 3H\varpi. \quad (8)$$

The above equation can be interpreted as a balance equation for the energy density of the gravitational field. By comparing equations (6) and (8) we conclude that the non-equilibrium pressure ϖ is the responsible for the irreversible transfer of energy between the gravitational and radiation fields.

The relationship between the energy density of the Chaplygin gas and the cosmic scale factor follows from the integration of (5) by considering the equation of state (1) and reads

$$\rho_c = \sqrt{A + \frac{B}{a^6}}, \quad (9)$$

where B is a constant of integration.

We determine the time evolution of the cosmic scale factor from the Friedmann equation (4) by differentiating it with respect to time, yielding

$$\dot{H} + 2H^2 = 4\pi G \left(\frac{4A + B/a^6}{3\sqrt{A + B/a^6}} - \varpi \right). \quad (10)$$

Equation (10) is a function of the non-equilibrium pressure and we assume that it is a variable within the framework of extended (causal or second-order) thermodynamic theory. In this case the evolution equation for the non-equilibrium pressure – in a linearised theory – reads (for a derivation of this equation from a microscopic point of view see, for example [22])

$$\varpi + \tau \dot{\varpi} = -3\eta H, \quad (11)$$

where τ is a characteristic time while η is the coefficient of bulk viscosity.

The system of equations (10) and (11) is closed by assuming that the coefficient of bulk viscosity η and the characteristic time τ are related to the energy densities by [17, 16]

$$\eta = \alpha(\rho_c + \rho_r), \quad \tau = \frac{\eta}{(\rho_c + \rho_r)}, \quad (12)$$

where α is a constant.

In order to find the solution of the system of equations (10) and (11) it is convenient to write it in a dimensionless form. To this end we introduce the dimensionless quantities

$$H \equiv \frac{H}{H_0}, \quad t \equiv tH_0, \quad a \equiv a \left(\frac{A}{B} \right)^{1/6}, \quad \alpha \equiv \alpha H_0, \quad \varpi \equiv \frac{8\pi G \varpi}{3H_0^2}, \quad (13)$$

where the index zero denotes the value of the quantity at $t = 0$ (by adjusting clocks). The Hubble parameter H_0 is given in terms of the energy densities of the Chaplygin gas and radiation at $t = 0$, namely

$$H_0 = \sqrt{\frac{8\pi G}{3}(\rho_c^0 + \rho_r^0)}. \quad (14)$$

With respect to the above dimensionless quantities the equations (10) and (11) read

$$\dot{H} + 2H^2 = \frac{3}{2} \left(\frac{1}{3(1 + \rho_r^0/\rho_c^0)\sqrt{1 + 1/a_0^6}} \frac{4 + 1/a^6}{\sqrt{1 + 1/a^6}} - \varpi \right), \quad (15)$$

$$\varpi + \alpha \dot{\varpi} = -3\alpha H^3. \quad (16)$$

The system of differential equations (15) and (16) are used to determine the cosmic scale factor $a(t)$ and the non-equilibrium pressure $\varpi(t)$ once three initial conditions for $a(0)$, $\dot{a}(0)$ and $\varpi(0)$ are specified and values are given for the parameter α (which is connected with the irreversible processes) and for the ratio ρ_r^0/ρ_c^0 (which gives the order of magnitude between the energy densities of the Chaplygin gas and radiation at $t = 0$). From the knowledge of the time evolution of the cosmic scale factor $a(t)$ one can determine the time evolution of the energy densities of the Chaplygin gas $\rho_c(t)$ and of the radiation $\rho_r(t)$, which follow from (9) and (4), respectively,

$$\rho_c = \frac{\sqrt{1 + 1/a^6}}{\sqrt{1 + 1/a_0^6}}, \quad \rho_r = \frac{\rho_c^0}{\rho_r^0} \left[\left(1 + \frac{\rho_r^0}{\rho_c^0} \right) H^2 - \frac{\sqrt{1 + 1/a^6}}{\sqrt{1 + 1/a_0^6}} \right], \quad (17)$$

where $\rho_c \equiv \rho_c/\rho_c^0$ and $\rho_r \equiv \rho_r/\rho_r^0$ are dimensionless quantities.

If we consider the Universe as a mixture of quintessence and radiation, the dimensionless equations (15) and (17) are replaced, respectively, by [16]

$$\dot{H} + 2H^2 = \frac{3}{2} \left[\frac{(1 - 3w_X)}{3(1 + \rho_r^0/\rho_X^0)} \left(\frac{1}{a} \right)^{3(w_X+1)} - \varpi \right], \quad (18)$$

$$\rho_X = \left(\frac{1}{a} \right)^{3(w_X+1)}, \quad \rho_r = \left(1 + \frac{\rho_X^0}{\rho_r^0} \right) H^2 - \frac{\rho_X^0}{\rho_r^0} \left(\frac{1}{a} \right)^{3(w_X+1)}, \quad (19)$$

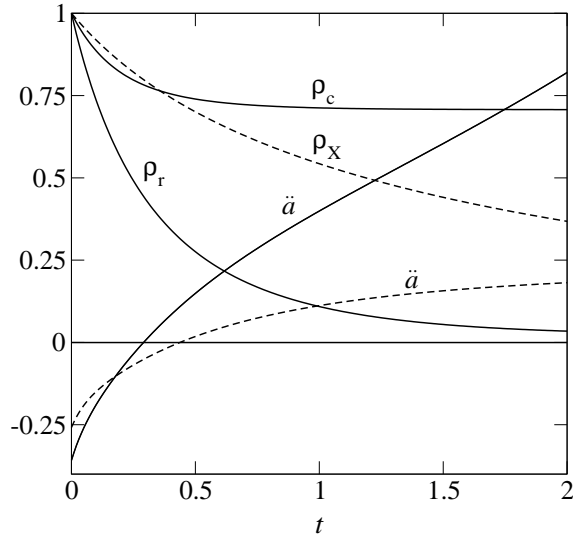


Figure 1: Energy densities of Chaplygin gas ρ_c , of quintessence ρ_X and of radiation ρ_r vs time t . Acceleration \ddot{a} vs time t for Chaplygin gas and radiation (straight line), for quintessence and radiation (dashed line).

while the evolution equation for the non-equilibrium pressure (16) remains unchanged. In the above equations ρ_X is the quintessence energy density which has an equation of state $p_X = w_X \rho_X$. Moreover, according to [6, 4, 5], the parameter w_X must satisfy the condition $w_X < -1/3$.

To solve the system of equations (15) and (16) for the mixture of the Chaplygin gas and radiation, as well as the corresponding system (18) and (16) for the mixture of quintessence and radiation, we specify the following initial conditions: $a(0) = 1$ for the cosmic scale factor and $H(0) = 1$ for the Hubble parameter and $\tilde{\omega}(0) = 0$ for the non-equilibrium pressure. There still remains much freedom to find the solutions of these two systems of differential equations, since they do depend on some parameters. For both mixtures we choose the parameter $\alpha = 0.05$ (say). By decreasing the value of α the influence of the dissipative effects is less pronounced. The other parameter is the ratio ρ_r^0/ρ_c^0 (or ρ_r^0/ρ_X^0) which gives the amount of the energy density of radiation with respect to the energy density of the Chaplygin gas (or quintessence) at $t = 0$. We assume that $\rho_r^0/\rho_c^0 = \rho_r^0/\rho_X^0 = 2$ (say). There exists one more parameter for the case of a mixture of quintessence and radiation, namely w_X . We consider $w_X = -0.7$ in order to satisfy the condition $w_X < -1/3$.

In Fig. 1 we have plotted the energy density of the Chaplygin gas ρ_c , the energy density of the radiation ρ_r and the energy density of the quintessence ρ_X as functions of the time t . Moreover, in this figure the acceleration \ddot{a} is represented as a function of the time t by a straight line for the mixture of a Chaplygin gas with radiation, while by a dashed line for the mixture of quintessence with radiation. From this figure we infer that for both models there exists a period of a past deceleration with a present acceleration. The slope of the acceleration of the Universe modelled as a mixture of a Chaplygin gas with radiation is more pronounced than that modelled as a mixture of quintessence and radiation. Further, the energy density of the Chaplygin gas tends to a constant value at earlier times than the energy density of quintessence does. The energy density of radiation for both mixtures coincide and decays more rapidly than the energy densities of the Chaplygin gas and of quintessence. The same conclusion as in the work [16] can be drawn here: even when a small amount of energy density of the Chaplygin gas (or quintessence) – with respect to the energy density of radiation – is taken into account, these fields evolve in such a manner that for large times the energy density of the Chaplygin gas (or quintessence) is very large with respect to the energy density of radiation. Other conclusions that can be drawn here are similar to those found in the work [16]: (a) in the case where there is no energy density of the Chaplygin gas (or quintessence) only a period of deceleration is possible, i.e., there exists no accelerated period; (b) by decreasing the value of the dimensionless constant α the effect of the non-equilibrium pressure is less pronounced and the period of past deceleration increases.

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